Cichocki, A., Zdunek, R., Phan, A. H., and Amari, S. Nonnegative Matrix and Tensor Factorizations: Applications to Exploratory Multi-Way Data Analysis and Blind Source Separation. John Wiley \& Sons Ltd., Chichester West Sussex, United Kingdom, 2009.

Coifman, R. R. and Lafon, S. Diffusion maps. Applied and Computational Harmonic Analysis, 21(1):5-30, July 2006.

Colombo, A., Cusano, C., and Schettini, R. 3D face detection using curvature analysis. Pattern Recognition, 39(3):444-455, March 2006.

Comon, P. Independent component analysis, a new concept? Signal Processing, 36(3):287-314, 1994.

Comon, P. and Mourrain, B. Decomposition of quantics in sums of powers of linear forms. Signal Processing, 53:93-108, 1996.

Comon, P., Luciani, X., and De Almeida, A.L.F. Tensor decompositions, alternating least squares and other tales. Journal of Chemometrics, 23(7-8):393-405, 2009.

Cootes, T. F., Edwards, G. J., and Taylor, C. J. Active appearance models. IEEE Transactions on Pattern Analysis and Machine Intelligence, 23(6): 681-685, June 2001.

Cristianini, N. and Shawe-Taylor, J. An Introduction to Support Vector Machines and Other Kernel-Based Learning Methods. Cambridge University Press, United Kingdom, 2000.

Cunado, D., Nixon, M. S., and Carter, J. N. Automatic extraction and description of human gait models for recognition purposes. Computer Vision and Image Understanding, 90(1):1-41, January 2003.

Dai, G. and Yeung, D. Y. Tensor embedding methods. In Proc. Twenty-First National Conference on Artificial Intelligence, pp. 330-335, July 2006.

Dalal, N. and Triggs, B. Histograms of oriented gradients for human detection. In Proc. IEEE Conference on Computer Vision and Pattern Recognition, volume 1, pp. 886-893, 2005.

Daubechies, I. The wavelet transform, time-frequency localization and signal analysis. IEEE Transactions on Information Theory, 36(5):961-1005, 1990.

De la Torre, F. A least-squares framework for component analysis. IEEE Transactions on Pattern Analysis and Machine Intelligence, (34):10411055, 2012.

De Lathauwer, L. Signal Processing Based on Multilinear Algebra. PhD thesis, Katholieke Universiteit Leuven, 1997. URL ftp://ftp.esat.kuleuven. ac.be/sista/delathauwer/reports/PHD.pdf.

He, X., Yan, S., Hu, Y., Niyogi, P., and Zhang, H. Face recognition using Laplacianfaces. IEEE Transactions on Pattern Analysis and Machine Intelligence, 27(3):328-340, March 2005b.

Hillis, D. M. and Bull, J. J. An empirical test of bootstrapping as a method for assessing confidence in phylogenetic analysis. Systematic Biology, 42(2): 182-192, 1993.

Hinton, G.E. and Salakhutdinov, R.R. Reducing the dimensionality of data with neural networks. Science, 313(5786):504-507, 2006.

Hinton, G.E., Osindero, S., and Teh, Y.W. A fast learning algorithm for deep belief nets. Neural Computation, 18(7):1527-1554, 2006.

Hitchcock, F. L. The expression of a tensor or a polyadic as a sum of products. Journal of Mathematical Physics, 6(1):164-189, 1927a.

Hitchcock, F. L. Multiple invariants and generalized rank of a p-way matrix or tensor. Journal of Mathematical Physics, 7(1):39-79, 1927b.

Ho, T. K. The random subspace method for constructing decision forests. IEEE Transactions on Pattern Analysis and Machine Intelligence, 20(8): 832-844, August 1998.

Höskuldsson, A. PLS regression methods. Journal of Chemometrics, 2(3): 211-228, 1988.

Hotelling, H. Analysis of a complex of statistical variables into principal components. Journal of Educational Psychology, 24(6):417-441, 1933.

Hotelling, H. Relations between two sets of variables. Biometrika, 28(3/4): 312-377, 1936.

Howe, D., Costanzo, M., Fey, P., Gojobori, T., Hannick, L., Hide, W., Hill, D. P, Kania, R., Schaeffer, M., St. Pierre, S., et al. Big data: The future of biocuration. Nature, 455(7209):47-50, 2008.

Hsu, R.-L., Abdel-Mottaleb, M., and Jain, A. K. Face detection in color images. IEEE Transactions on Pattern Analysis and Machine Intelligence, 24(5):696-706, 2002.

Hua, G., Viola, P. A., and Drucker, S. M. Face recognition using discriminatively trained orthogonal rank one tensor projections. In Proc. IEEE Conference on Computer Vision and Pattern Recognition, pp. 1-8, June 2007.

Hyvärinen, A. Fast and robust fixed-point algorithms for independent component analysis. IEEE Transactions on Neural Networks, 10(3):626-634, May 1999.

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